OCR

RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Subsidiary (AS) and Advanced Level

MATHEMATICS **M1**

Mechanics 1

Additional materials:
Answer paper
Graph paper
List of Formulae

SPECIMEN PAPER

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.

You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

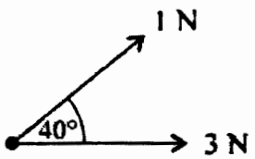
The total number of marks for this paper is 60.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

1



Two forces, of magnitudes 1 N and 3 N, act on a particle in the directions shown in the diagram. Calculate the magnitude of the resultant force on the particle and the angle between this resultant force and the force of magnitude 3 N. **[5]**

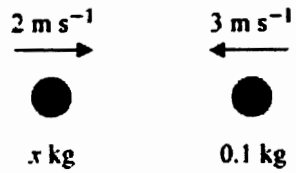
2

The diagram shows a railway engine of mass 50 tonnes pulling two trucks horizontally along a straight track. The trucks are coupled together behind the engine and have masses 8 tonnes and 4 tonnes respectively, starting with the truck nearer to the engine. The acceleration of the train is $0.5\,\mathrm{m\,s^{-2}}$. Assuming that there are no resistances to motion, find

(i) the driving force of the engine, **[2]**

(ii) the tensions in the two couplings. **[4]**

3



Two particles, of masses x kg and 0.1 kg, are moving towards each other in the same straight line and collide directly. Immediately before the impact, the speeds of the particles are $2\,\mathrm{m\,s^{-1}}$ and $3\,\mathrm{m\,s^{-1}}$ respectively (see diagram).

(i) Given that both particles are brought to rest by the impact, find x. **[2]**

(ii) Given instead that the particles move with equal speeds of $1\,\mathrm{m\,s^{-1}}$ after the impact, find the three possible values of x. **[6]**

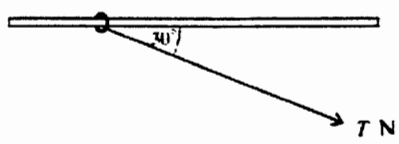
4 A moving particle P travels in a straight line. At time t seconds after starting from the point O on the line, the velocity of P is $v\,\mathrm{m\,s^{-1}}$, where

$$v = t^2(6-t).$$

Show that the acceleration of P is zero when $t = 4$. **[3]**

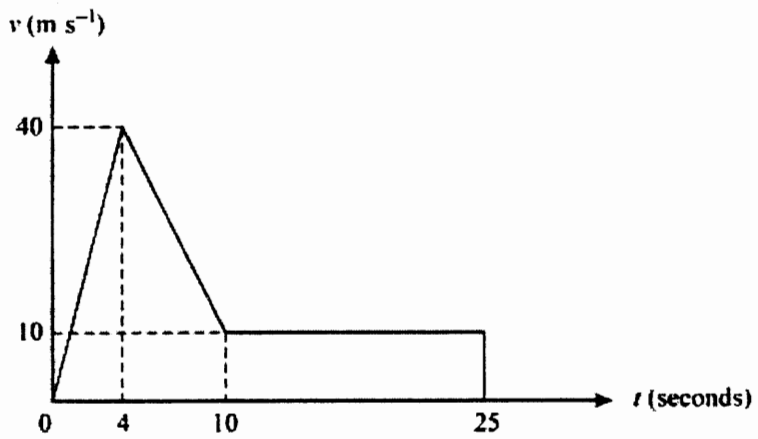
After a certain time, P comes instantaneously to rest at the point A on the line. State the time taken for the motion from O to A, and find the distance OA. **[5]**

5



A heavy ring of mass 5 kg is threaded on a fixed rough horizontal rod. The coefficient of friction between the ring and the rod is $\frac{1}{2}$. A light string is attached to the ring and is pulled downwards with a force of magnitude T newtons acting at an angle of 30° to the horizontal (see diagram). Given that the ring is about to slip along the rod, find the value of T. **[9]**

6



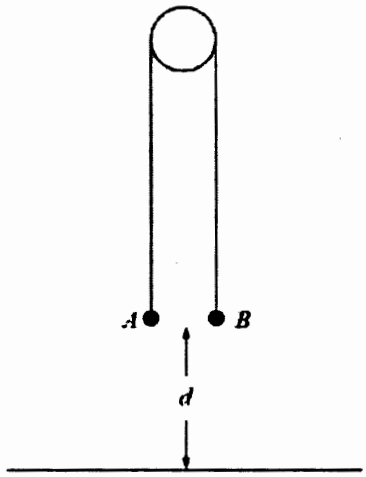
The diagram shows an approximate (t, v) graph for the motion of a parachutist falling vertically; v m s^{-1} is the parachutist's downwards velocity at time t seconds after he jumps out of the plane. Use the information in the diagram

(i) to give a brief description of the parachutist's motion throughout the descent, **[4]**

(ii) to calculate the height from which the jump was made. **[2]**

The mass of the parachutist is 90 kg. Calculate the upwards force acting on the parachutist, due to the parachute, when $t = 7$. **[5]**

7 (i)



Particles A, of mass $5m$, and B, of mass $3m$, are attached to the ends of a light inextensible string. The string passes over a fixed peg, and the system is released from rest with both parts of the string taut and vertical, and each particle a distance d above a fixed horizontal plane (see diagram). Neglecting all resistances to motion,

(a) find the acceleration of A in terms of g and show that the tension in the string is $\frac{15}{4}mg$, **[6]**

(b) find an expression in terms of d and g for the time after release at which A hits the plane. **[2]**

(ii) The results in part **(i)** are based on a mathematical model in which resistances to motion are neglected. Describe briefly one resisting force, other than air resistance, which would be present in a real system in which objects of unequal mass, hanging from a string passing over a fixed support, are in motion. **[1]**

When this force is taken into account, state with brief reasons whether each of the following would be smaller or larger than the value calculated in part **(i)**:

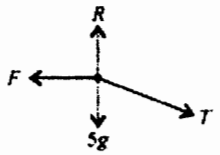
(a) the acceleration of A;

(b) the tension in the string acting on A;

(c) the tension in the string acting on B.

What can you conclude about the tension in the string in this case? **[4]**

1	*EITHER* : 'Vertical' component of resultant is $1\sin 40°$	B1		
	'Horizontal' component is $3+1\cos 40°$	B1		
	Magnitude is $\sqrt{3.766^2+0.6428^2}$	M1		Allow M mark for either Pythagoras or trig
	i.e. 3.82 N	A1✓		For correct magnitude
	Angle is $\tan^{-1}\left(\frac{0.6428}{3.766}\right)=9.69°$	A1✓		For correct angle
	OR : Vector triangle with sides 1, 3 and included angle of 140° (not 40°)	B1		May be implied
	$R^2=3^2+1^2-2\times3\times1\times\cos140°$	M1		For use of cosine formula with 3, 1, 140°
	Hence magnitude is 3.82 N	A1✓		
	$\frac{\sin\theta}{1}=\frac{\sin140°}{3.82}$	M1		For sin formula, or other complete method
	Hence angle is 9.69°	A1✓	5	
2	(i) Force $=(50000+8000+4000)\times0.5$	M1		For use of NII applied to whole system
	$=31000$ N or 31 kN	A1	2	
	(ii) For back truck: $C_1=4000\times0.5$	M1		Use of NII for the rear truck only
	i.e. Force in rear coupling is 2000 N or 2 kN	A1		
	For both trucks: $C_2=12000\times0.5$	M1		Use of NII for the pair of trucks with one force, or equivalent, i.e. $C_2-C_1=4000$ or $31000-C_2=25000$
	i.e. Force in front coupling is 6000 N or 6 kN	A1	4	Follow through if earlier answer is used
3	(i) $x\times2-3\times0.1=0$	M1		For relevant use of momentum conservation
	Hence $x=0.15$	A1	2	
	(ii) $2x-0.3=(x+0.1)$ or $-(x+0.1)$ or $0.1-x$	M1		For any one relevant momentum equation
		A1		For any one correct (unsimplified) equation
		M1		For appreciating at least 2 correct cases
	Hence $x=0.4$ or 0.0667 or 0.133	A1		For any one correct value
		A1		For a second correct value
		A1	6	For all three correct answers
4	$\frac{dv}{dt}=12t-3t^2=0$	M1		For expanding v and differentiating
		A1		For correct derivative equated to zero
	$3t(4-t)=0$, so $a=0$ when $t=4$	A1	3	Given answer correctly found or verified
	P reaches A when $t=6$	B1		
	$s=\int_0^6(6t^2-t^3)\,dt=\left[2t^3-\frac{1}{4}t^4\right]_0^6$	M1		For integrating v
		A1		For correct indefinite integral
	$=432-324$	M1		Use of limits or evaluation of arbitrary const
	Distance $OA=108$ m	A1✓	5	

5		B1		Correct forces identified, by diagram or otherwise
	Resolving horizontally: $T\cos 30° = F$	M1 A1		For attempting one resolution equation
	Resolving vertically: $5g + T\sin 30° = R$	M1 A1		For attempting a second resolution Other correct equations are possible
	For limiting equilibrium $F = \frac{1}{2}R$	B1		Available at any stage
	$T.\frac{1}{2}\sqrt{3} = \frac{1}{2}(5g + \frac{1}{2}T)$	M1		For eliminating F and R
		A1✓		Correct unsimplified equation in T only
	$T = 39.8$	A1✓	9	
6	(i) Initially the parachutist falls with constant acc Then decelerates at a constant rate Then falls with constant speed And finally hits the ground and comes to rest	B1 B1 B1 B1	4	Allow 'free-fall' etc here
	(ii) Area is $\frac{1}{2}\times 4\times 40 + \frac{1}{2}(40+10)\times 6 + 10\times 15$ Height is 380 m	M1 A1	2	For sensible attempt at total area under graph
	Acceleration when $t = 7$ is $\frac{10-40}{10-4} = -5$ (downwards)	M1 A1		For use of gradient to find acceleration For value (±)5 even if sign/direction muddle
	Hence $90g - T = 90\times(-5)$	M1 B1		For use of NII with three relevant terms For consistent signs in T and ma terms
	Force from parachute is 1330 N	A1	5	
7	(i) (a) Equations of motion for the particles are: $5mg - T = 5ma$ $T - 3mg = 3ma$ Hence acceleration is $\frac{1}{4}g$ and Tension is $\frac{15}{4}mg$	M1 A1 A1 M1 A1 A1	6	For use of NII for either particle separately The 'system' equation $8mg = 2ma$ is an alternative for one of these A marks For finding T or a from sufficient equation(s) For correct acceleration For obtaining given tension correctly
	(b) $d = \frac{1}{2}\times\frac{1}{4}g\times t^2$ Time is $\sqrt{\frac{8d}{g}}$	M1 A1✓	2	Use appropriate *uvast* equation and solve for t
	(ii) Friction between the string and the support	B1	1	
	(a) Acceleration is smaller, as the resistance opposes the motion (b) Tension at A is larger, because $T_A = 5mg - 5ma$, and a is less than before (c) Tension at B is smaller, because $T_B = 3ma + 3mg$ and a is less than before The tensions in the two parts of the string are now unequal	B1 B1 B1 B1	4	